

International Conference on Research in Natural and Engineering Sciences (ICRNES 2020)

Cylindrically Symmetric String Cloud with Quark Matter in Unimodular f(R) Gravity

(Hüseyin Aydın, Melis Ulu Doğru)

Cylindrically Symmetric String Cloud with Quark Matter in Unimodular f(R) Gravity

Hüseyin Aydın[1], Melis Ulu Doğru[2]

[1]*Çanakkale Onsekiz Mart University, School of Graduate Studies, Çanakkale/TURKEY, E-mail: huseynaydin@gmail.com*
[2]*Çanakkale Onsekiz Mart University, Faculty of Arts and Sciences, Department of Physics, Çanakkale/TURKEY, E-mail: melisulu@comu.edu.tr*

Abstract: *In this study, we aim to examine cylindrically symmetric string cloud with quark matter in unimodular f(R) gravity. f(R) denotes a general function of Ricci scalar in Einstein-Hilbert action. Lagrange multiplier imposes unimodularity in action integral. Determinant of the spacetime metric tensor is set to a constant in unimodular gravitational theories. We use unimodular and static cylindrically symmetric spacetime. Unimodular form of the spacetime is considered an appropriate coordinate transformation. String cloud is a particle connected cosmic string which is one-dimensional topological defect in early universe. Components of the energy momentum tensor are obtained. Field equations are written down. Their solutions for given matter and the spacetime are found. By using an ansatz, arctangential f(R) function is achieved. Graphics of the string tension density and total energy density of string cloud are depicted with respect to radial coordinate (r). It is found that string tension density has a singularity at r = 0. Quark matter density approaches to bag constant (vacuum energy that is surrounding the quark) while radial coordinate goes to infinity. Total energy density becomes zero at r = 0 and r → +∞. Also, it is determined maximum point of total energy density. We get Lagrange multiplier by using the trace of the field equations. Obtained results interpreted by using f(R) models, boundary conditions and matter properties. We show that arctangential f(R) models can be applied in unimodular f(R) gravity. In addition, we concluded that string cloud consisting of quark matter added cosmic string can be exist in unimodular f(R) gravity context.*

Key Words: *unimodular f(R) gravity; string cloud; quark matter.*

Introduction

Unimodular f(R) gravity is one of the alternative gravitation theories to general relativity [1-3]. In this theory, f(R) represents a general function of Ricci scalar in Einstein's general relativity [1-3]. Unimodularity constrains the determinant of spacetime metric tensor to a constant value such as $\sqrt{-g} = \epsilon$ [1]. In unimodular f(R) gravity, a Langrange multiplier λ is included in the f(R) action function [2].

$$S = \frac{1}{2\kappa}\int\left[\sqrt{-g}\left(f(R)-\lambda\right)+\lambda\right]d^4x + S_m \qquad (1)$$

S_m indicates matter Lagrangian. Unimodular f(R) gravity field equations are defined by

$$\frac{1}{2}g_{ik}(f(R)-\lambda) - R_{ik}U(R) + \nabla_i\nabla_k U(R) - g_{ik}\nabla^2 U(R) + \frac{1}{2}T_{ik} = 0 \qquad (2)$$

where $U(R) = df(R)/dR$, ∇_i: covariant derivative, ∇^2: d'Alembert operator, T_{ik}: energy-momentum tensor. Trace of the Eq. (2) can be written as

$$2(f(R) - \lambda) - R\,U(R) - 3\nabla^2 U(R) + \frac{1}{2}T = 0 \qquad (3)$$

where $T = g^{ik}T_{ik}$ is the trace of the energy-momentum tensor. Eliminating $f(R) - \lambda$ by using Eq. (2) and Eq. (3) gives rearranged unimodular f(R) gravity field equations [1-3]:

$$\frac{1}{2}g_{ik}R\,U(R) - R_{ik}U(R) + \nabla_i\nabla_k U(R) - \frac{1}{4}g_{ik}\nabla^2 U(R) + \frac{1}{2}T_{ik} - \frac{1}{8}g_{ik}T = 0. \qquad (4)$$

In early epoch of the Universe, some symmetry breakings causes topological defects [4]. Main topological defects are monopoles, cosmic strings, domain walls and textures [4]. String clouds can be defined particle added strings. In this work, we use quark matter added string cloud. The energy momentum tensor can be written as follow [5]:

$$T_i^k = (\rho_s + \rho_q + B_c)u_i u^k - \rho_s X_i X^k \qquad (5)$$

where B_c: vacuum energy density (bag constant), ρ_q: quark matter density, ρ_s: string tension density, u_i: 4-velocity and X_i: direction vector with $X_i X^i = 1$. Cylindrically symmetric spacetime is given by [6,7]

$$ds^2 = e^{2B(r)}dr^2 + r^2 d\phi^2 + dz^2 - e^{2A(r)}dt^2. \qquad (6)$$

Unimodular And Cylindrically Symmetric String Cloud With Quark Matter

In order to obtain the unimodular spacetime metric, a coordinate transformation such as $d\tau = re^{A(r)}e^{B(r)}dt$ can be taken into account. Substituting this transformation into the metric gives unimodular cylindrically symmetric spacetime as below.

$$ds^2 = e^{2B(r)}dr^2 + dz^2 + r^2 d\phi^2 - \frac{1}{e^{2B(r)}r^2}d\tau^2 \qquad (7)$$

Energy-momentum tensor components are obtained as

$$T_1^1 = -\rho_s(r),\ T_4^4 = -\rho_q(r) - \rho_s(r) - B_c. \qquad (8)$$

Unimodular f(R) gravity field equations can be written down as follows in the presence of cylindrically symmetric string cloud with quark matter. ($'$) indicates derivative with respect to radial coordinate (r).

$$6U'' - 4(B'U)' + 8(B')^2 U + e^{2B}(\rho_q - 2\rho_s + B_c) + 12\frac{U}{r}\left(B' + \frac{1}{r}\right) = 0, \qquad (9)$$

$$e^{-2B}\left(-(B')^2U+\frac{1}{2}(B'U)'-\frac{3}{2}\frac{U}{r^2}-\frac{1}{4}U''-\frac{1}{r}\left(\frac{5}{2}B'U-U'\right)\right)+\frac{1}{8}\left(\rho_q+2\rho_s+B_c\right)=0, \quad (10)$$

$$e^{-2B}\left(-8(B')^2U+4(B'U)'-4\frac{U}{r^2}-2U''-\frac{4}{r}B'U\right)+\rho_q+2\rho_s+B_c=0, \quad (11)$$

$$e^{-2B}\left(-8(B')^2U+4(B'U)'-4\frac{U}{r^2}+2U''-\frac{1}{r}(12B'U-8U')\right)+3\rho_q+2\rho_s+3B_c=0. \quad (12)$$

Solutions of the field equations give $B(r)$, $\rho_q(r)$ and $\rho_s(r)$ functions. c_1 denotes integration constant.

$$B(r)=-\frac{1}{2}\ln(r)+\frac{1}{2}\ln(U(r))+c_1, \quad (13)$$

$$\rho_s(r)=\frac{2}{e^{2B(r)}r^2}(2(B')^2Ur^2-(B'U)'r^2+U''r^2+rU'+U), \quad (14)$$

$$\rho_q(r)=-\frac{1}{e^{2B(r)}r^2}\left(-8(B')^2Ur^2+e^{2B(r)}r^2(2\rho_s+B_c)+4(B'U)'r^2-4B'Ur-2U''r^2-4U\right). \quad (15)$$

If we take into account ansatz $U(r)=\frac{1}{1+r^2}$ and assume $c_1=0$, Eqs. (13)-(15) become as followings.

$$B(r)=\frac{1}{2}\ln\left(\frac{1}{r(r^2+1)}\right), \quad (16)$$

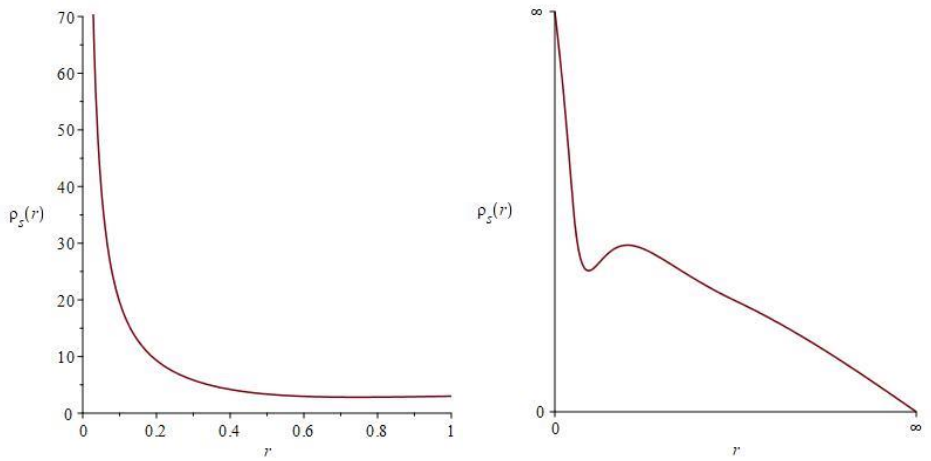
$$\rho_s(r)=\frac{10r^4+2}{r(r^2+1)^2}, \quad (17)$$

$$\rho_q(r)=-B_c-\frac{r^2(10r^2-4)-2}{r(r^2+1)^2}. \quad (18)$$

Total energy density of string cloud with quark matter can be written as

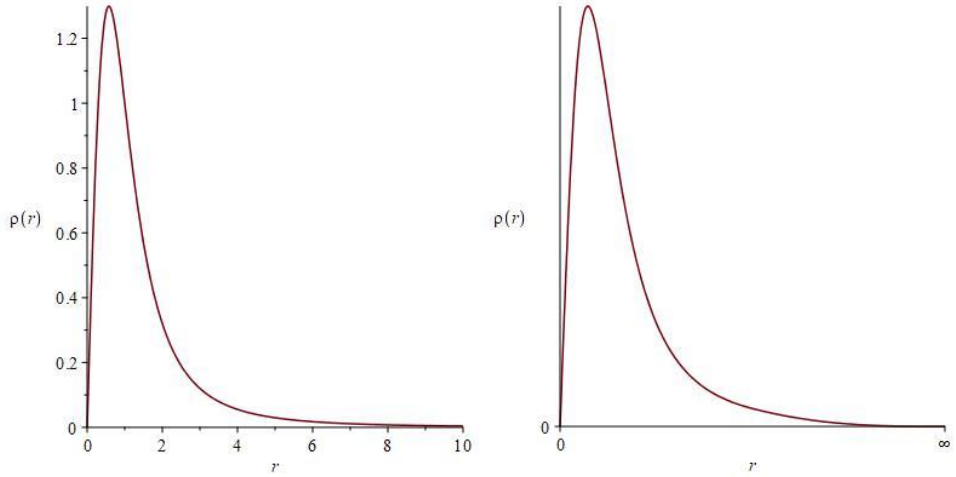
$$\rho=\rho_s+\rho_q+B_c=\frac{4r}{(r^2+1)^2}. \quad (19)$$

Graphs of the ρ_s and ρ can be depicted as follows.



(a) (b)

Figure 1. String tension density with respect to (r), a) $0 \leq r \leq 1$, b) $0 \leq r < +\infty$.



(a) (b)

Figure 2. Total energy density of string cloud with respect to (r), a) $0 \leq r \leq 10$, b) $0 \leq r < +\infty$.

Ricci scalar obtained as

$$R(r) = \frac{-5\,r^2-1}{r}\,. \tag{20}$$

In order to derive $f(R)$ function, we substitute the inverse function of Ricci scalar $r(R)$ into $U(r) = \frac{1}{1+r^2}$ function.

$$U(R) = \frac{50}{R^2+R\sqrt{R^2-20}+40}\,. \tag{21}$$

Integrating $U(R)$ with respect to R, we obtain $f(R)$ function as

$$f(R) = \frac{R}{2} - \frac{\sqrt{R^2-20}}{2} + 3\,arctan\left(\frac{4\sqrt{R^2-20}+6R}{24-R\sqrt{R^2-20}}\right) + c_2. \quad (22)$$

Also, Lagrange multiplier may be shown as

$$\lambda(r) = f(R(r)) + 2\,\rho(r). \quad (23)$$

Conclusion

We study cylindrically symmetric string cloud with quark matter in unimodular f(R) gravity. Unimodular form of the spacetime is considered an appropriate coordinate transformation. The field equations and their solutions are written down. Energy densities and f(R) function are obtained.

Limit of total energy density in Eq. (19) at $r \to +\infty$ is zero. Also, energy density of quark matter ρ_q approaches to bag constant (vacuum energy enclosing the quark) at $r \to +\infty$. String tension density has singularity at $r = 0$. Total energy density becomes zero at $r = 0$ and has a maximum at r = √3 / 3. Total energy density and string tension density boundaries can be seen in figures 1-2.

Arctangential f(R) models proposed and discussed in Refs. [8,9] in the context of cosmological constant. In this study, we show that such arctangential models can be used in unimodular f(R) gravity. Also, we concluded that string cloud consisting of quark matter added string can be exist in unimodular f(R) gravity context.

Acknowledgments

This study has been supported by Scientific Research Projects Commission of Çanakkale Onsekiz Mart University (BAP) with the project no. FDK-2018-2730 and includes some part of Ph.D. thesis of Hüseyin Aydın.

References

Eichhorn, A. (2015). The Renormalization Group flow of unimodular f(R) gravity. *Journal of High Energy Physics*, *2015*(4), 96.

Nojiri, S., Odintsov, S. D., & Oikonomou, V. K. (2016). Unimodular F(R) gravity. *Journal of Cosmology and Astroparticle Physics*, *5*(046), 1512-07223.

Nojiri, S., Odintsov, S. D., & Oikonomou, V. K. (2016). Bounce universe history from unimodular F(R) gravity. *Physical Review D*, *93*(8), 084050.

Vilenkin A. and Shellard E. P. S. (1994). *Cosmic Strings and Other Topological Defects* (Cambridge University Press, Cambridge, 1994), pp. 54-84.

Yavuz, I., Yilmaz, I., & Baysal, H. (2005). Strange quark matter attached to the string cloud in the spherical symmetric space–time admitting conformal motion. *International Journal of Modern Physics D*, *14*(08), 1365-1372.

Chao-Guang, H. (1995). Charged static cylindrical black hole. *Acta Physica Sinica (Overseas Edition)*, *4*(8), 617.

Sharif, M., & Arif, S. (2012). Static cylindrically symmetric interior solutions in f (R) gravity. *Modern Physics Letters A*, *27*(25), 1250138.

Kruglov, S. I. (2014). Modified arctan-gravity model mimicking a cosmological constant. *Physical Review D*, *89*(6), 064004.

Kruglov, S. I. (2015). Arctan-gravity model. *Universe*, *1*(1), 82-91.